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Weakly coupled Higgsless theories and precision electroweak tests

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Abstract

In 5 dimensions the electroweak symmetry can be broken by boundary conditions, leading to a new type of Higgsless theories. These could in principle improve on the 4D case by extending the perturbative domain to energies higher than $4\pi v$ and by allowing a better fit to the electroweak precision tests. Nevertheless, it is unlikely that both these improvements can be achieved, as we show by discussing these problems in an explicit model.

1 Introduction

Higgsless theories of the electroweak interactions do not appear to allow an acceptable description of the ElectroWeak Precision Tests (EWPT) [1]. At least, since they become strongly interacting at an energy of about $4\pi v$, where v is the Higgs vacuum expectation value, the calculability of the precision observables is limited. Furthermore, when an estimate can be made under suitable assumptions [2], the ultraviolet (UV) contribution to the parameter ϵ_3 [3] (or to the parameter S [4]) is positive, which, added to the infrared (IR) piece cut off at some high scale $\sim 4\pi v$, makes it essentially impossible to fit the EWPT, independently of the value of ϵ_1 (or of the T parameter).

Breaking the electroweak symmetry by boundary conditions on an extra dimension may however be a new twist of the problem (see for example refs. [5, 6]). The specific reason for this statement is the following. It is well known that a gauge group G can be broken by boundary conditions at compactification down to a subgroup H. In this situation the vectors in H have a Kaluza-Klein (KK) tower starting with a massless 4D mode, while the lightest state in G/H has mass $1/R$, where R is the radius of compactification. A feature of gauge symmetry breaking by boundary conditions, for example on orbifolds, is that in general it does not introduce new physical scales associated to a strongly interacting regime in the theory. For instance, using naive dimensional analysis (NDA)[7, 8], a five dimensional gauge theory will become strongly coupled at a scale $\Lambda \sim 24\pi^3/g_5^2$, whether it is broken by boundary conditions or not. Now the potentially interesting fact is that, if we interpret the lightest KK modes in G/H as the W and Z, the 5D cut-off Λ is written in terms of 4D quantities as $\Lambda \sim 12\pi^2 m_W/g_4^2$ (we have used $1/g_4^2 = 2\pi R/g_5^2$ and $m_W \sim 1/R$). Compared to the cut-off $4\pi m_W/g_4 \sim 1$ TeV of a 4D Higgsless theory, the cut-off of the 5D Higgsless theory is a factor $3\pi/g_4$ bigger: $\Lambda \sim 10$ TeV. To summarize, what happens physically is the following: an appropriate tower of KK states may play the role of the normal 4D Higgs boson in preventing the relevant amplitudes from exceeding the unitarity bounds up to an energy scale Λ which is not arbitrarily large but can be well above the cut-off $4\pi v$ of a 4D Higgsless theory. This is why we call them weakly interacting Higgsless theories.

One may do the same power counting exercise for a D-dimensional gauge theory compactified down to 4-dimensions. In that case the cut-off is $\Lambda^{D-4} \sim (4\pi)^{D/2}\Gamma(D/2)/g_D^2$, while

the 4D gauge coupling is $1/g_4^2 = (2\pi R)^{D-4}/g_D^2$. If we identify $m_W \sim 1/R$ we can then write $\Lambda = m_W[16\Gamma(D/2)\pi^{(4-D/2)}/g_4^2]^{1/(D-4)}$, from which we conclude that by going to $D > 5$ we do not actually increase the cut-off. For $D = 6$ the cut-off is again roughly $4\pi v$, while for $D \to \infty$ it becomes $\sim m_W$. This result is intuitively clear: with a large number of extra dimensions, in order to keep g_4 finite, the radius of compactification should be right at the cut-off. We therefore stick to $D = 5$.

The next obvious problem is to see how these 5D Higgsless theories can do in describing the EWPT, which is the purpose of this paper *. We do this by analysing a specific model designed to keep under control the effects of the breaking of custodial isospin, so that we can focus on the effects of ϵ_3 only. We shall also comment on the likely general validity of our conclusions.

2 The model

Motivated by the simple argument given in the introduction we consider a 5D gauge theory compactified on S_1/Z_2. Since we want calculable (and small) custodial symmetry breaking effects, we must separate in 5D the sectors that break the electroweak symmetry from those that break the custodial symmetry. A necessary requirement to achieve this property is the promotion of custodial symmetry to a gauge symmetry. Then the minimal model with a chance of giving a realistic phenomenology has gauge group $G = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Following refs. [10, 11] we break $G \to SU(2) \times U(1)_Y$ at one boundary, $y = 0$, and $G \to SU(2)_{L+R} \times U(1)_{B-L}$ at the other boundary, $y = \pi R$. Then the only surviving gauge symmetry below the compactification scale is $U(1)_Q$. We achieve the breaking by explicitly adding mass terms for the broken generator vector at the boundary and by sending this mass to infinity †. The most general lagrangian up to two derivatives is then

$$\mathcal{L} = \mathcal{L}_5 + \delta(y)\mathcal{L}_0 + \delta(y - \pi R)\mathcal{L}_\pi \,, \tag{1}$$

*For a related discussion, see ref. [9].

†Even if the boundary mass is kept to its natural value, $M \sim \Lambda$, this will only modify our calculations by $O(M_L/(\Lambda^2 R))$ effects.

with

$$\begin{aligned}
\mathcal{L}_5 &= -\frac{M_L}{4}(W_L^I)_{MN}(W_L^I)^{MN} - \frac{M_R}{4}(W_R^I)_{MN}(W_R^I)^{MN} - \frac{M_B}{4}B_{MN}B^{MN}\,, \qquad (I=1,2,3)\\
\mathcal{L}_0 &= -\frac{1}{4g^2}(W_L^I)_{\mu\nu}(W_L^I)^{\mu\nu} - \frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} - \frac{M^2}{2}\left[(W_R^+)_\mu(W_R^-)^\mu + (W^3_{R\mu}-B_\mu)(W_R^{3\,\mu}-B^\mu)\right]\,,\\
\mathcal{L}_\pi &= -\frac{Z_W}{4}(W_L^I)_{\mu\nu}(W_L^I)^{\mu\nu} - \frac{Z_B}{4}B_{\mu\nu}B^{\mu\nu} - \frac{M^2}{2}(W^I_{R\,\mu}-W^I_{L\,\mu})(W_R^{I\,\mu}-W_L^{I\,\mu})\,.
\end{aligned} \tag{2}$$

In the limit $M\to\infty$ at $y=0$ we have $W_R^+=0$ and $W_R^3=B$, then the kinetic terms in $\mathcal{L}_0$ are truly the most general ones. A similar comment applies at $y=\pi R$.

Notice that, although we have added extra operators at the boundaries, the UV cut-off of this theory is still determined by the bulk gauge couplings. It is intuitively clear why the large mass terms at the boundaries do not lower the cut-off: they can be viewed as originating from spontaneous gauge symmetry breaking by a σ-model sector with typical scale $\sim M$. Moreover the kinetic terms at the boundaries, when they are large and positive, just make some degrees of freedom more weakly coupled at the boundary (When they are large and negative they lead to light tachyonic ghosts; see discussion below). An explicit proof that boundary masses plus kinetic terms are innocuous could be obtained by using the technique of ref. [12]. It would correspond to choosing an analogue of 't Hooft-Feynman gauge where both the σ-model Goldstones and the 5th components of the vectors at the boundaries (which are normally set to zero when working on an orbifold) are kept non-zero. In such a gauge, unlike in the unitary gauge, the propagators are well behaved in the UV and the loop power counting straightforward. [‡] Therefore, according to NDA, we define the strong coupling scales of the bulk gauge theories by

$$\Lambda_i \equiv 24\pi^3 M_i\,, \qquad i=L,R,B\,. \tag{3}$$

Finally we assume the SM fermions to be (in first approximation) localized at $y=0$, away from the source of electroweak breaking. In this way, extra unwanted non-oblique corrections are kept at a minimum. We will later comment on how to give fermions a mass while keeping the suppression of extra four-fermion interactions.

[‡]In ref. [6] it was shown that an infinite mass at the boundary does not lower the unitarity cut-off of elastic KK scattering. Inelastic processes were however not studied.

3 The low energy effective theory

To study the low energy phenomenology one way to proceed is to find the KK masses and wave functions. However, since the SM fermions couple to $W_L^I(x, y = 0) \equiv \bar{W}^I(x)$ and $W_R^3(x, y = 0) = B(x, y = 0) \equiv \bar{B}(x)$, it is convenient to treat the exchange of vectors in a two step procedure. First we integrate out the bulk to obtain an effective lagrangian for $\bar{W}^I$ and $\bar{B}$. Then we consider the exchange of the interpolating fields $\bar{W}^I$ and $\bar{B}$ between light fermions. Indeed we will not need to perform this second step: to compare with the data we just need to extract the ϵ's [3], or S, T, U observables [4], from the effective lagrangian for the interpolating fields. This way of proceeding is clearly inspired by holography, though we do not want to emphasize this aspect for the time being.

To integrate out the bulk, we first must solve the 5D equations of motions, imposing at $y = \pi R$ the boundary conditions that follow from the variation of the action, while at $y = 0$ the fields are fixed at $W_L^I(x, y = 0) = \bar{W}^I(x)$ and $W_R^3(x, y = 0) = B(x, y = 0) = \bar{B}(x)$. By substituting the result back into the action, we obtain the 4D effective lagrangian

$$\mathcal{L}_{eff} = \int_0^{2\pi R} dy \left(\mathcal{L}_5 + \mathcal{L}_\pi\right) . \tag{4}$$

We will work in the unitary gauge gauge where the 5th components of the gauge fields are set to zero. At the quadratic level in the 4D fields $\bar{W}_L^I(x), \bar{B}(x)$, integration by parts and use of the equations of motion allows to write $\mathcal{L}_{eff}$ as a boundary integral

$$\mathcal{L}_{eff} = \left[M_L \bar{W}_\mu^I \partial_y W_L^{I\,\mu} + \bar{B}_\mu \left(M_R \partial_y W_R^{3\,\mu} + M_B \partial_y B^\mu\right)\right]_{y=0} . \tag{5}$$

In order to solve the equations of motion it is useful to work in the momentum representation along the four non-compact dimensions: $x_\mu \to p_\mu$. Moreover it is useful to separate the fields in the longitudinal and transverse components $V_\mu(y) = V_\mu^t + V_\mu^l$ satisfying separate 5D equations

$$\begin{aligned} (\partial_y^2 - p^2) V_\mu^t &= 0 \,, \\ \partial_y^2 V_\mu^l &= 0 \,. \end{aligned} \tag{6}$$

Nevertheless, since we are interested in the coupling to light fermions, we will just focus on the transverse part and eliminate the superscript alltogether. The general bulk solution

has the form $V_\mu = a_\mu \cosh(py) + b_\mu \sinh(py)$ where a_μ and b_μ are fixed by the boundary conditions. After a straightforward computation we find

$$\mathcal{L}_{eff} = \bar{W}^I_\mu \Sigma_L(p^2) \bar{W}^{I\mu} + \bar{W}^3_\mu \Sigma_{3B}(p^2) \bar{B}^\mu + \bar{B}_\mu \Sigma_{BB}(p^2) \bar{B}^\mu \,, \tag{7}$$

with

$$\begin{aligned} \Sigma_L &= -M_L \frac{2M_L\, p \tanh(p\pi R) + 2M_R\, p \coth(p\pi R) + Z_W\, p^2}{2(M_L + M_R) + Z_W\, p \tanh(p\pi R)} \,, \\ \Sigma_{3B} &= -\frac{4M_L M_R\, p(\tanh(p\pi R) - \coth(p\pi R))}{2(M_L + M_R) + Z_W\, p \tanh(p\pi R)} \,, \\ \Sigma_{BB} &= -M_R \frac{2M_R\, p \tanh(p\pi R) + 2M_L\, p \coth(p\pi R) + Z_W\, p^2}{2(M_L + M_R) + Z_W\, p \tanh(p\pi R)} \\ & \quad -M_B \frac{2M_B\, p \tanh(p\pi R) + Z_B\, p^2}{2M_B + Z_B\, p \tanh(p\pi R)} \,. \end{aligned} \tag{8}$$

The total lagrangian $\mathcal{L}_0 + \mathcal{L}_{eff}$ gives us the complete effective theory as a function of the boundary fields at $y = 0$. The KK spectrum of the model and the couplings of the KK modes to the boundaries can be obtained by finding the poles and residues of the full inverse kinetic matrix. It is instructive (and also phenomenologically preferable, as we will discuss) to consider the limiting case where the boundary kinetic terms at $y = 0$ dominate the contribution from the bulk: $1/g^2, 1/g'^2 \gg M_i \pi R, Z_{W,B}$. In this limit the physical masses sit very close to the poles of the Σ/p^2's. For instance, for the charged vectors we have two towers of modes that we call odd and even [§] . In the limit $Z_W = 0$ the odd modes are

$$m_{n+1/2} = \frac{n + \frac{1}{2}}{R} \left[1 + \frac{2g^2 M_L^2 R}{(n + \frac{1}{2})^2 (M_L + M_R)} + \ldots \right] \qquad n = 0, 1, 2, ... \tag{9}$$

and the even ones are

$$\begin{aligned} m_n &= \frac{n}{R} \left[1 + \frac{2g^2 M_L M_R R}{\pi n^2 (M_L + M_R)} + \ldots \right] \qquad n > 0 \,, \\ m_0^2 &= \frac{2g^2 M_L M_R}{\pi (M_L + M_R) R} \left[1 + O(g^2 M_{L,R} R) \right] \equiv m_W^2 \,. \end{aligned} \tag{10}$$

[§] Notice that when $M_L = M_R$ the bulk theory is invariant under parity $W^L \leftrightarrow W^R$, and the odd and even modes are respectively vector and axial under parity.

In the limit we are considering, the lightest mode is much lighter than the others $m_0 \ll 1/R$: it sits close to the Goldstone pole of Σ_L/p^2. This mode should be interpreted as the usual W-boson of the standard model. Similarly in the neutral sector we find a lightest massive boson, the Z, with mass

$$m_Z^2 = \frac{2(g^2+g'^2)M_L M_R}{\pi(M_L+M_R)R}\left[1+O(g^2 M_{L,R}R)\right] . \tag{11}$$

One can compute the couplings of Z, W and photon to elementary fermions from the first ∂_{p^2} derivative of the full 2-point function at the zeroes. For small $g^2 M_{L,R}R$, one finds that the leading contribution arises from the boundary lagrangian $\mathcal{L}_0$. We then conclude that, up to corrections of $\mathcal{O}(g^2 M_{L,R}R)$, the SM relations between masses and couplings are satisfied. This can also be seen from the wave-function of the lowest KK modes that, for large kinetic terms on the $y=0$ boundary, is peaked at $y=0$, and then the SM gauge fields correspond approximately to the boundary fields $\bar{W}^I$ and $\bar{B}$.

All the deviation from the SM at the tree level are due to oblique corrections and can be studied by expanding eq. (8) at second order in p^2 around $p^2=0$: $\Sigma(p^2)\simeq\Sigma(0)+p^2\Sigma'(0)$. As custodial isospin is manifestly preserved by eq. (7), we find that the only non-zero observable is ϵ_3 (S parameter):

$$\epsilon_3 = -g^2\Sigma'_{3B}(0) = g^2\frac{4\pi R}{3}\frac{M_L M_R}{M_L+M_R}\left[1+\frac{3Z_W}{4\pi(M_L+M_R)R}\right] . \tag{12}$$

It is convenient to define $1/\Lambda = 1/\Lambda_R + 1/\Lambda_L$, so that Λ is essentially the cut-off of the theory. It is also convenient to define $Z_W \equiv \delta/16\pi^2$, as $\delta = O(1)$ corresponds to the natural NDA minimal size of boundary terms. This way ϵ_3 is rewritten as

$$\epsilon_3 = \frac{g^2}{18\pi^2}(\Lambda R)\left[1+\frac{9\delta}{8(\Lambda_R+\Lambda_L)R}\right] . \tag{13}$$

Now, the loop expansion parameter of our theory is $\sim 1/(\Lambda R)$. In order for all our approach to be any better that just a random strongly coupled electroweak breaking sector we need $\Lambda R \gg 1$. This can be reconciled with the experimental bound $\epsilon_3 \lesssim 3\cdot 10^{-3}$ ¶ only if δ

¶ $\epsilon_3 \lesssim 3\cdot 10^{-3}$ is the limit on the extra contribution to ϵ_3 relative to the SM one with $m_H = 115$ GeV. The limit is at 99% C.L. for a fit with ϵ_1 and ϵ_3 free, but ϵ_2 and ϵ_b fixed at their SM values. Letting ϵ_2 and ϵ_b be also free would weaken the limit, but only in a totally marginal way.

is large and negative, $\delta \sim -\Lambda R$, and partially cancels the leading term in eq. (13). This requirement, however, leads to the presence of a tachyonic ghost with $m^2 \sim -1/R^2$ in the vector spectrum, again a situation that would make our effective theory useless.

4 Fermion masses

The discussion so far assumed the SM fermions to be exactly localized at $y = 0$, in which case they would be exactly massless, having no access to the electroweak breaking source at $y = \pi R$. As we have argued, in this limiting case there are only oblique corrections to fermion interactions: all the information that there exists an extra dimension (and some strong dynamics) is encoded in the vector self-energies in eq. (7). For example, there are no additional 4-fermion contact interactions.

A more realistic realization of fermions is the following. Consider bulk fermions with the usual quantum numbers under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. For instance, the right handed fermions will sit in $(1, 2, B-L)$ doublets. Each such 5D fermion upon orbifold projection will give rise to one chiral multiplet with the proper quantum numbers under $SU(2)_L \times U(1)_Y$. Fermion mass operators mixing left and right multiplet can then be written at $y = \pi R$, very much like the $W_R - W_L$ vectors. However in the case of fermions, since their mass dimension in 5D is 2, the mass coefficient at the boundary is dimension-less. Then, in the limit in which the scale of electroweak breaking at $y = \pi R$ is sent to infinity, the fermion mass at the boundary should stay finite. Indeed if we indicate by F the scale of electroweak breaking at the boundary and put back the Goldstone field matrix U which non linearly realizes the symmetry, the fermion mass operator has the form $\bar{\psi} U \psi$ while the vector mass arises from the Goldstone kinetic term $F^2 (D_\mu U)^\dagger (D^\mu U)$.

Finally, to make the masses small, to break isospin symmetry, and to cause effective approximate localization on the $y = 0$ boundary it is enough to add large kinetic terms for the fermions at $y = 0$. Since isospin is broken at $y = 0$ the kinetic terms will distinguish fermions with different isospin, for instance s_R from c_R, and a realistic theory can be easily obtained. Fermion masses m_f will go roughly like $1/\sqrt{Z_L Z_R}$, where Z_L and Z_R are the boundary kinetic coefficients of the left and right handed components respectively. Notice that the Z's for fermions have dimension of length. Non oblique effects due to the fermion

tail into the bulk will then scale like $\sim R/Z \sim m_f/m_{KK}$, which is negligible for the 2 light generations, but probably not for the bottom quark. One could also spread the fermions more into the bulk by decreasing the $Z's$ while decreasing at the same time the fermion mass coefficients at the electroweak breaking boundary. This way one would get more sizeable non-oblique corrections to EWPT, in the form of corrections to the W and Z vertices and four-fermion interactions from the direct coupling to vector KK modes. In principle these extra parameters could be used to improve the fit, tuning the effective ϵ_3 small. We do not pursue this here, since we do not think of this possibility as being very compelling.

5 ϵ_3 for a general metric

One can wonder whether different 5D geometries can change the result above by giving, for example, a negative contribution to ϵ_3. Here we will show that this is not the case and ϵ_3 stays positive whatever the metric. As we want to preserve 4D Poincaré symmetry the curvature will just reduce to a warping. It is convenient to choose the 5th coordinate y in such a way that the metric is

$$ds^2 = e^{2\sigma(y)} dx^\mu dx_\mu + e^{4\sigma(y)} dy^2 \,, \tag{14}$$

and take, to simplify the notation, $0 \leq y \leq 1$ ($\pi R = 1$). With this choice the bulk equation of motion of a transverse vector field becomes

$$(\partial_y^2 - p^2 e^{2\sigma}) V_\mu = 0 \,. \tag{15}$$

To simplify the discussion we limit ourselves to the case $M_R = M_L = M$, in which parity is conserved in the bulk and we set $Z_W = 0$. Moreover, as $U(1)_{B-L}$ does not play any role in ϵ_3, we neglect it alltogether. The extension to the general case is straightforward.

To calculate ϵ_3 it is convenient to work in the basis of vector $V = (W_L + W_R)/\sqrt{2}$ and axial $A = (W_L - W_R)/\sqrt{2}$ fields, for which

$$\epsilon_3 = -\frac{g^2}{4} [\Sigma'_V(0) - \Sigma'_A(0)] \,, \tag{16}$$

where $\Sigma_V = 4MV^{(-1)}\partial_y V|_{y=0}$ and similarly for Σ_A. Since we are only interested in the Σ's at $O(p^2)$, we can write the solutions of the bulk equations of motion as

$$\begin{aligned} V_\mu &= \bar{V}_\mu(p^2)\left(v^{(0)}(y) + p^2 v^{(1)}(y) + O(p^4)\right) , \\ A_\mu &= \bar{A}_\mu(p^2)\left(a^{(0)}(y) + p^2 a^{(1)}(y) + O(p^4)\right) , \end{aligned} \tag{17}$$

where, as before, $\bar{A}_\mu$ and $\bar{V}_\mu$ are the fields at $y = 0$. The functions $v^{(0)}$ and $a^{(0)}$ solve eq. (15) at $p^2 = 0$, with boundary conditions $v^{(0)}(0) = a^{(0)}(0) = 1$ and $\partial_y v^{(0)}|_{y=1} = a^{(0)}(1) = 0$. We obtain

$$v^{(0)} = 1\,, \qquad a^{(0)} = 1 - y\,. \tag{18}$$

The function $v^{(1)}$ solves $\partial_y^2 v^{(1)} = e^{2\sigma(y)} v^{(0)}$ with boundary conditions $v^{(1)}(0) = \partial_y v^{(1)}|_{y=1} = 0$. For $a^{(1)}$ the boundary conditions are $a^{(1)}(0) = a^{(1)}(1) = 0$. Then we find

$$\epsilon_3 = g^2 M \left\{ \int_0^1 e^{2\sigma(y)} dy \; - \; \int_0^1 dy \int_0^y (1 - y') e^{2\sigma(y')} dy' \right\} , \tag{19}$$

which is manifestly positive. The first term can in fact be written as an integral in two variables by multiplying by $1 = \int_0^1 dy'$. Then, since $e^{2\sigma} > (1-y)e^{2\sigma} > 0$ and the domain of integration of the second term is a subset of the domain of the first, $\epsilon_3 > 0$ follows.

While ϵ_3 is always positive, it could become very small if $e^{2\sigma}$ decreases rapidly away from $y = 0$. However this is the situation in which the bulk curvature becomes large. Indeed the Ricci scalar is given by $\mathcal{R} = [8\sigma'' + 4(\sigma')^2]e^{-4\sigma}$ and tends to grow when the warp factor decreases. In fact the curvature length scales roughly like $e^{2\sigma}$, so that the general expression in eq. (19) roughly corresponds to the flat case result, eq. (13), with the radius R replaced by the curvature length at $y = 1$.

As an explicit example consider the family of metrics (in the conformal frame)

$$ds^2 = (1 + y/L)^{2d} \left\{ dx_\mu dx^\mu + dy^2 \right\} , \tag{20}$$

with $0 \le y \le \pi R$ as originally. With this parametrization the massive KK modes are still quantized in units of $1/R$. On the other hand the curvature goes like

$$\mathcal{R} \sim \frac{1}{L^2(1 + y/L)^{2+2d}} , \tag{21}$$

so that for $d < -1$ it grows at the electroweak breaking boundary ($d = -1$ corresponds to AdS). Indicating $\Delta = (1 + \pi R/L)$ we have

$$\epsilon_3 = g^2 M \frac{L}{d+1} \left\{ -1 + \Delta^{1+d} + \frac{4 + (d-1)^2(\Delta^2 - 1) + (d-3)\Delta^{1-d} - (1+d)\Delta^{d-1}}{(3-d)(2 - \Delta^{1-d} - \Delta^{d-1})} \right\} , \quad (22)$$

which for $d < -1$, $L \ll R$ (big warping) gives roughly $\epsilon_3 \simeq g^2 \Lambda R_C/(18\pi^2)$ where the curvature length at $y = 1$, $R_C \sim L\Delta^{d+1}$, has replaced the radius R. Of course when $\Lambda R_C < 1$ we loose control of the derivative expansion for our gauge theory. We then conclude that the model in the regime of calculability, $\Lambda R, \Lambda R_C \gg 1$, gives always large contributions to ϵ_3.

6 Similarities with strongly coupled 4D theories

In this Section we comment on the relation between the model presented here and technicolor-like theories in 4D. We will consider the case $M = M_L = M_R$ and $Z_W = 0$.

As done for the case of AdS/CFT one can establish a qualitative correspondence between the bulk theory (in fact the bulk plus the $y = \pi R$ boundary) and a purely four dimensional field theory with a large number of particles, indeed a large N theory. The loop expansion parameter $1/(\Lambda R)$ of the 5D theory corresponds to the topological expansion parameter $1/N$, so that our tree level calculation corresponds to the planar limit. Consistent with this interpretation, the couplings among the individual KK bosons go like $1/\sqrt{N}$, as expected in a large N theory. The result of eq. (13), written as $\epsilon_3 \sim g^2 N/18\pi^2$, also respects the correspondence: it looks precisely like what one would expect in a large N technicolor. Similarly the 1-loop gauge contribution to ϵ_1 and ϵ_2 is proportional to $g'^2/16\pi^2$, with no N enhancement. This result is easy to understand in the "dual" 4D theory. Custodial isospin is only broken by a weak gauging of hypercharge by an external gauge boson ($\equiv$ living at the $y = 0$ boundary). Isospin breaking loop effects involve the exchange of this single gauge boson and are thus not enhanced by N. Similar considerations can be made for the top contribution. Of course, though useful, this correspondence is only qualitative, in that we do not know the microscopic theory on the 4D side. We stress that this qualitative correspondence is valid whatever the metric of the 5D theory. The case of AdS geometry only adds conformal symmetry into the game allowing for an (easy) extrapolation (for a subset of observables) to arbitrarily high energy. On the other hand, when looking for solutions to

the little hierarchy problem [13], one can be content with a theory with a fairly low cut-off (maybe 10 TeV) in which case conformal symmetry is not essential.

The correspondence with a large N technicolor also goes through for the sign of ϵ_3. Very much like S is positive in rescaled versions of QCD [14, 2], we have proven a positive S theorem for a class of "holographic" technicolor theories. Is there a simple reason for this relation? Perhaps some insight can be obtained by realizing that the $\Sigma_{V,A}$, both in our 5D models and in a generic large N theory, can be rewritten as a sum over narrow resonances

$$\Sigma_V = -p^2 \sum_n \frac{F_{V_n}^2}{p^2 + m_{V_n}^2}\,, \qquad \Sigma_A = -p^2 \sum_n \frac{F_{A_n}^2}{p^2 + m_{A_n}^2} - f_\pi^2\,. \tag{23}$$

Then, from eq. (16) we have

$$\epsilon_3 = \frac{g^2}{4} \sum_n \left[\frac{F_{V_n}^2}{m_{V_n}^2} - \frac{F_{A_n}^2}{m_{A_n}^2} \right]\,. \tag{24}$$

For a flat extra dimension we have $F_{V_n}^2 = F_{A_n}^2 = 8M/(\pi R)$, $f_\pi^2 = 4M/(\pi R)$, and the masses $m_{V_n} = (n + 1/2)/R$ and $m_{A_n} = (n+1)/R$ with $n = 0, 1, 2, \ldots$. Then ϵ_3 is dominated by the first resonance, a vector, and turns out positive. Qualitatively, what happens is the following: vector and axial resonances alternate in the spectrum and, since the lightest state is a massless Goldstone boson in the axial channel, the lightest massive state tends to be a vector, so that ϵ_3 tends to be positive.

In ref. [2] positive ϵ_3 was deduced by saturating eqs. (23) and (24) with the first two low laying $J = 1$ resonances, called ρ and a_1 mesons, after imposing the two Weinberg sum rules [15]:

$$\epsilon_3^{TC} = \frac{g^2}{4} \left(1 + \frac{m_\rho^2}{m_{a_1}^2} \right) \frac{f_\pi^2}{m_\rho^2}\,. \tag{25}$$

In our 5D model $\Sigma_V - \Sigma_A$ vanishes exponentially in the large euclidean momentum region, and then an infinite set of generalized Weinberg sum rules are satisfied [‖]. However all levels are involved in the sum rules, so that, strictly speaking, one cannot play rigorously the same game. There are, nevertheless, some surprising numerical coincidences. For example, in flat 5D, we have

$$m_\rho = \frac{1}{2R}\,, \qquad m_{a_1} = \frac{1}{R}\,, \tag{26}$$

[‖] These arise by imposing that the coefficients of the Taylor expansion of $\Sigma_V - \Sigma_A$ (from eq. (23)) at $p^2 \to \infty$ are zero.

and eq. (12) can be rewritten as

$$\epsilon_3 = \frac{g^2\pi^2}{30}\left(1+\frac{m_\rho^2}{m_{a_1}^2}\right)\frac{f_\pi^2}{m_\rho^2}\,. \tag{27}$$

This result deviates by less than a 30% from the expression of eq. (25).

Based on these considerations, one is therefore driven to establish a connection, inspired by holography, between the 5D model presented here and technicolor-like theories in the large N limit. If this is the case, the impossibility to fit the EWPT, while keeping the perturbative expansion, goes in the same direction as the claimed difficulty encountered in 4D technicolor-like theories to account for the EWPT.

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